An instrumental orchestration for online course at the university level

Una orquestación instrumental para un curso en línea a nivel universitario

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Keywords
Instrumental orchestration; dynamic geometry environment; linear algebra; mathematics education; online course

ABSTRACT

In this article, we present a proposal for instrumental orchestration that organizes the use of technological environments in online mathematics education, in the synchronous mode for the concepts of eigenvalue and eigenvector of a first linear algebra course with engineering students. We used the instrumental orchestration approach as a theoretical framework to plan and organize the artefacts involved in the environment (didactic configuration) and the ways in which they are implemented (exploitation modes). The activities were designed using interactive virtual didactic scenarios, in a dynamic geometry environment, guided exploration worksheets with video and audio recordings of the work of the students, individually or in pairs. The results obtained are presented and the orchestrations of a pedagogical sequence to introduce the concepts of eigenvalue and eigenvector are briefly discussed. This work allowed us to identify new instrumental orchestrations for online mathematics education.

Palabras clave
Orquestación instrumental; entorno de geometría dinámica; álgebra lineal; educación matemática; enseñanza en línea

RESUMEN

En este artículo se presenta una propuesta de orquestación instrumental, la cual organiza el uso de los entornos tecnológicos en la enseñanza de la matemática en línea (modalidad sincrónica) para los conceptos de valor y vector propio de un primer curso de álgebra lineal con estudiantes de ingeniería. Se utilizó el enfoque de la orquestación instrumental como marco teórico para planificar y organizar los artefactos que intervienen en el entorno (configuración didáctica) y las formas en las que se implementan (modo de explotación). Las actividades se diseñaron mediante escenarios didácticos virtuales interactivos, en un entorno de geometría dinámica, hojas de exploración guiadas y videograbaciones del trabajo de manera individual o por pares de los estudiantes. Se presentan los resultados obtenidos y se discuten las orquestaciones de una secuencia de instrucción para introducir los conceptos de valor y vector propio. El trabajo permitió identificar nuevas orquestaciones instrumentales para la enseñanza de la matemática en línea.

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INTRODUCTION

During March 2020, the World Health Organization (WHO) declared a global pandemic due to Covid-19, which caused educational institutions around the world to migrate from face-to-face to online instruction. Most institutions were not prepared for this change, so teachers had to get involved in the use and management of digital technologies; likewise, students had to adapt quickly to this new online teaching modality. This brought serious problems in education (Engelbrecht et al., 2020). This article offers a contribution in the investigation of the teaching and learning of linear algebra in a virtual environment through the orchestration of various digital artifacts or devices, which is aimed at university students. For years, work has been done on the design and construction of educational software to support the teaching of mathematics (Cuevas-Vallejo & Mejía, 2003; Orozco-Santiago, 2014; 2020). Due to this, it was possible to adapt to this new teaching in a short time, from which the experience achieved in a linear algebra course is presented.

Linear algebra is one of the first abstract mathematics courses that students encounter in the initial years in higher education (Oktaç & Trigueros, 2010; Stewart, Andrews-Larson, Berman, & Zandieh, 2018). The extensive use of rigorous definitions and the proof of theorems and lemmas make this subject one of the most formal and abstract in the engineering mathematics curriculum and, consequently, one of the subjects with the highest rate of student failure and frustration (Carlson et al., 1993; Dorier, 2000; Orozco-Santiago, 2020).

Research has been carried out from different theoretical perspectives to study the obstacles faced by students in the teaching-learning process of linear algebra concepts. Among these we can mention Hillel (2000), who considered three levels of specific description languages (abstract, algebraic and geometric) of basic objects and operations; Sierpinska (2000), who points out three modes of thought: synthetic-geometric, analytic-arithmetic and analytic-structural; Dorier and Sierpinska (2001), who distinguish two inseparable sources of students' difficulties in learning and knowledge processes: the nature of linear algebra (conceptual difficulties) and the type of thinking necessary for its understanding (cognitive difficulties). On the other hand, the linear algebra curriculum study group (Carlson et al., 1993) recommended that mathematics teachers be encouraged to use technology in the first linear algebra course: "We believe that students' use of computers for assignments and projects can reinforce classroom concepts, contribute to the discovery of new concepts, and make realistic applied problem solving feasible (p. 45)." To situate this work, background information is presented that informs instructional materials to introduce and promote students' understanding of eigenvalues and eigenvectors.

LITERATURE REVIEW

The current digital era induces dramatic changes in the way teachers and students access information and construct knowledge, in the way they communicate, interact, and work (Artigue, 2016). The National Council of Teachers of Mathematics (NCTM, 2000) argues that "technology is essential in the teaching and learning of mathematics; it influences the mathematics being taught and reinforces student learning" (p. 24). In the 1980s, the use of scientific calculators became popular, however, these were not designed for educational purposes (their design was largely forced by the available technology) and the initial targets of their sales were commercial and scientific work (Monaghan, Trouche, & Borwein, 2016). Computer Algebra Systems (CAS) because of their graphical, symbolic and numerical capabilities have been more exploited in differential and integral calculus than in linear algebra.

Linear algebra is one of the most abstract and formal subjects in higher education and is therefore cognitively and conceptually difficult (Dorier and Sierpinska, 2001). In addition, it has few resources for online teaching. In Khan Academy, Coursera and MéxicoX there are linear algebra courses, which due to their qualities do not consider its formal part. Both situations make it difficult to adapt them to online teaching.

One of the compulsory topics contained in the school curriculum in a first course of linear algebra is eigenvalues and eigenvectors, a topic that is taught at the end of the course and where most of the previous concepts are used. Because of this, the teaching-learning of these concepts has had problems in their acquisition by students. Some researchers have tried to facilitate their learning through the use of digital technologies. Klasa (2010) used two computational environments, Maple V and Cabri II, to study the concepts: linear transformation, eigenvalues and eigenvectors, quadratic forms, conics with changes of bases and singular values. Klasa provides an applet in the Maple environment to students, who run an animation of a unit vector υ rotating about a unit circle along with its image $T(v)$. The author asks them to observe at what point - if any - ν and $T(v)$ are collinear. This same scenario developed in Cabri provided students with interaction when manipulating the motion of the vector υ. Subsequently, using some Cabri tools, they measured the norms of ν and $T(v)$ and finally found the relation $|| T(v) ||/|| v ||$ that provides the associated eigenvalue. Klasa (2010) points out that Cabri facilitates geometric understanding and Maple supports by performing the matrix and algebraic-symbolic operations computationally.

With these experiences, the researcher states that visualization and manipulation enhance and facilitate the learning of linear algebra; furthermore, she adds that students working in teams around computers - or even graphing calculators - guided by the teacher, often become experts in the discipline they are experimenting with. In the same vein, Gol

Tabaghi (2014) studied the change of dynamic geometric representations in students' thinking by using different drag modalities in a dynamic geometry environment (The Geometer Sketchpad).

Gol Tabaghi (2014) conducts an analysis with three college students through individual clinical interviews based on tasks on the topic, of eigenvalues and eigenvectors, which the students had previously addressed. This mathematician designed several sketches to observe the geometry of the real eigenvalues and eigenvectors of a 2×2 matrix, which by dragging the vector $\vec{x}$ on the screen, the vector $\overrightarrow{Ax}$ moved accordingly, which attracted the students' attention to further analyze the relationship between $\vec{x}$ and $\overrightarrow{Ax}$ and, at the same time, make a coordination between the geometric representation in the sketch with the algebraic definition.

Supported by Sierpinska's (2000) three modes of thinking, Gol Tabaghi (2014) concludes that dynamic geometric representations allowed students to understand the concepts of vector and eigenvalue by identifying their invariant geometric properties and developing dynamic synthetic-geometric thinking; however, adding technology to a course may not necessarily bring about positive educational change (Drijvers et al., 2016; Hegedus et al., 2017).

THEORETICAL FRAMEWORK

Initially, the integration of technology consisted of giving it to students or teachers and explaining to them how to use it. This naive position was held in differential and integral calculus courses that used the Derive program, or in statistics courses where the spreadsheet was used. Even in the eighties it was thought that to teach geometry it was enough to introduce or show the basic operations of the turtle in Logo. When the desired results were not obtained, it was necessary to analyze how the different technological proposals could help in the teaching-learning of mathematics (Pantoja, 1997; Guin and Trouche, 2002).

Thus, proposals such as the instrumental approach emerged, which introduces a distinction between an artifact available (laptop, calculator, tablet or smartphone) to a learner when performing an activity (instrumentation process) and the conversion to an instrument during the course of an activity performed by him (instrumentalization process). This approach has been integrated into the didactics of mathematics (Rabardel, 1995; Artigue, 2002). Trouche (2004) introduces the term instrumental orchestration as a necessity for the teacher to organize interactions between students and instruments with particular didactic intentions; he presents two levels: a didactic configuration (arrangement of students and the artifacts available in the environment) and the modes of exploitation of these configurations.

The student-sherpa configuration (a student whose computer or calculator is projected in the classroom) was for several years an emblematic didactic configuration. For a given configuration, there are several possible modes of operation. Drijvers et al. (2010) consider, "[In] the musical metaphor of orchestration, the setting of the didactic configuration can be compared to the choice of musical instruments to be included in the orchestra and their arrangement in space so that the different sounds produce the most beautiful harmony (p. 3)." An instrumental orchestration is partially prepared beforehand and partially created "on the spot" during teaching, so Drijvers et al. (2010) add to the configurations and modes of exploitation a third level called didactic performance, which

> involves the most appropriate decisions to be made while teaching how to effectively realize the chosen didactic configuration and the mode of exploitation: what questions to ask, how to do justice (or leave aside) to any particular student input, how to deal with an unexpected aspect in the mathematical task or technological tool or other emergent objectives (p. 215, own translation). (p. 215, own translation).

In relation to the musical metaphor, Drijvers et al. (2010) suggest thinking of the triplet model as a jazz band, composed of beginning and advanced musicians (students), as well as the teacher, the band leader, who prepared a joint participation, but is open to student improvisation and interpretation, as well as doing justice to inputs at different levels. This statement became a reality in the present experience because there was no history of online teaching in linear algebra, so that teaching forms and methods had to be "improvised "[1]. Drijvers et al. (2010) extended the repertoire of instrumental orchestrations, and identified six types for whole-class teaching:

- Demonstration-technical. The teacher explains the technical aspects of using the tool. A didactic setup for this orchestration is an arrangement in the classroom in a way that allows students to see the projected screen and follow the demonstration.

- Explain-the-screen. The teacher's explanations go beyond the technical aspects and include mathematical content, guided by what appears on the screen. A didactic setup for this orchestration is a classroom arrangement in such a way that allows students to see the projected screen and follow the demonstration.

- Link-screen-blackboard. The teacher emphasizes to the class the connections between the representations on the screen and those of these mathematical concepts in books or on the blackboard. A didactic setup for this orchestration is an arrangement in the classroom in a way that allows students to see the projected screen, the writing on the blackboard, and follow the demonstration.

- Discuss-the-screen. The discussion about what is happening on the screen is led by the teacher in search of enhancing collective instrumental genesis. A didactic configuration for this orchestration is an arrangement in the classroom in a way that 1) allows the teacher access to the students' work and 2) the students can see the projected screen, what is written on the blackboard, follow the demonstration and encourage discussion.

- Indicate-and-display. The teacher identifies and displays the student work that he/she considers most relevant. A didactic setup for this orchestration is a classroom arrangement such that 1) it allows the teacher to access the students' work during lesson preparation, and 2) the students can view the projection of the work.

- Student-sherpa-in-the-work. The technological tool is in the hands of a student-sherpa, who will have the role of performing the activities. A didactic configuration for this orchestration is an arrangement in the classroom so that 1) it allows the student-sherpa to project his or her work or to perform the actions requested by the teacher, and 2) the students can see and follow the projection of the student-sherpa's work.

These orchestrations are not isolated, in the first three the teacher directs the communication, while in the last three the teacher gives more space to the students. In 2012, Drijvers et al. added a seventh orchestration: circulate-while-working, where students work on the computer alone or in pairs, while the teacher circulates between desks monitoring their progress and supporting them on technical or mathematical points. In 2013, Drijvers et al. refined the seventh orchestration and identified four additional types, as well as classifying them into two broad categories: whole-class orchestrations and individual or paired orchestrations.

Guide-and-explain. This is an intermediate orchestration between explain-the-screen and discuss-the-screen. A didactic configuration for this is a classroom arrangement that 1) allows the teacher to access the students' work, 2) allows the students to see the projected screen, see the writing on the board, and follow the demonstration, and 3) allows the teacher to ask questions-often closed-ended questions of the students.

Information-about-the-board. Depicts the teacher teaching and writing in front of the blackboard without technological support.

Demonstration-technical, guide-and-explain, link-screen-paper, discuss-the-screen, and support-technical. These are orchestrations in which students work individually or in pairs in front of their technological device. They have the didactic configuration in common, although they differ in their modes of exploitation.

Demonstrate-technique. The teacher demonstrates the techniques individually to avoid exposing the student to difficulties due to his or her inexperience with the digital environment.

Guide-and-explain. The teacher interacts with a student or a partner to explain or inform mathematical or technological aspects, based on what is shown on the screen.

Link-the-screen-and-paper. The teacher links what is shown on the screen with what is shown in books and links mathematics in a conventional pencil-and-paper way.

Discuss-the-screen. The teacher conducts the discussion with a student or with a partner, based on what is shown on the screen.

Support-technical. The teacher supports the student with technical problems, such as connection difficulties, software or hardware errors.

Nowadays, almost every didactic activity involves various actors, such as software, calculators, guides, books, the teacher, the student and the projector. It is necessary to establish the organization and management of the various artifacts or instruments involved in carrying out a given mathematical activity, as well as the role of the teacher, which maintains its importance. Thus, it is considered pertinent to pose the research question: what instrumental orchestrations might a university professor choose when using technology in online teaching of value and eigenvector?

METHODOLOGY

The teaching experiment was conducted with a group of 24 second year engineering students at a Mexican public university. Due to the pandemic, only ten students completed the course; there were five sessions lasting an hour and a half each; one of the authors was the professor of the course. Although the students had laptops or mobile devices (smartphones and tablets), the physical conditions necessary for the online classes were not entirely satisfactory. There were problems such as rather weak internet connections, noise both at home and outside, stress; a configuration of this is presented in Figure 1. The experience was developed through synchronous videoconferencing on the Zoom platform, in the same schedule as in the face-to-face classes. Most students opted not to use their cameras and microphones during the lectures given by the researcher, because a higher bandwidth was required to do so, and many did not have this access. The following forms of communication existed throughout the course: a) email, b) instant messaging (WhatsApp, proposed by the students), c) cloud storage service (Google Drive), and d) the files to be used in each session were uploaded to the cloud minutes before the class.

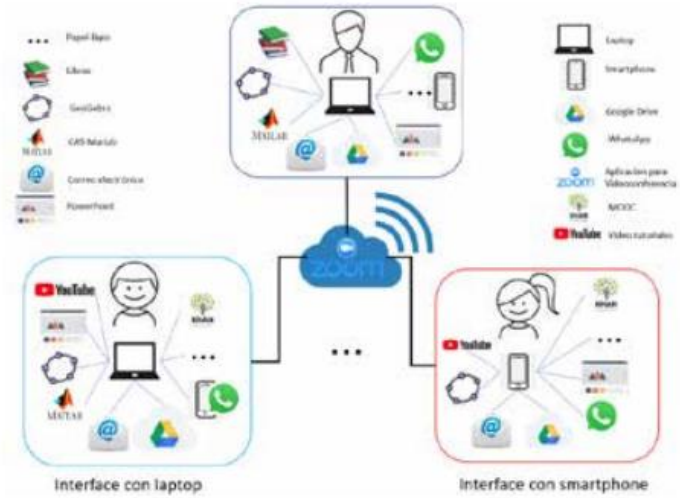

Figure 1. Participants in the experience and the configuration of artifacts used.

Data collection

Throughout the course, data were collected in the following ways: a) email messages; b) instant messaging via WhatsApp; c) videotaping of students' screens, for which the teacher proposed the free software OBS Studio in order to later deposit them in the cloud storage service (Google Drive); d) students' notes; e) video recordings of class activities that the researcher performed in Zoom; and f) videotaping of students' pair work which was also stored in the Google Drive folder. For the design of our task sequence, we used a didactic trajectory that was developed to be implemented in the classroom; however, due to the pandemic and confinement, this scheme was adapted to develop the activities online. The trajectory consists of seven student activities, organized by the teachers' instrumental orchestrations.

RESULTS

The instrumental orchestrations reported by Trouche (2004) and Drijvers et al. (2010 and 2013) have been developed for face-to-face classes in which the teacher and students are located in a classroom, face-to-face. Because of pandemic circumstances, this teaching experience was developed online synchronously, which allows some orchestrations to be used and others to be improvised.

In this experience several of the suggested orchestrations could not be applied; for example, circulate-while-working, because the teacher could not observe the real-time work of each of the students in this way (walking between computers) and see what they were doing. To cope with this situation there were two options: 1) ask the student for remote access to his or her computer or 2) have the student share his or her screen, which led to the development of mostly discuss-the-screen orchestration.

Figures 2a and 2b show a configuration used in the classroom: Figure 2a shows the PowerPoint open and the Zoom Annotation tool, and Figure 2b shows the interactive virtual teaching scenario (IVTS) and the Zoom Annotation tool enabled for writing on the screen; as can be seen, it is analogous to the usual classroom whiteboard.

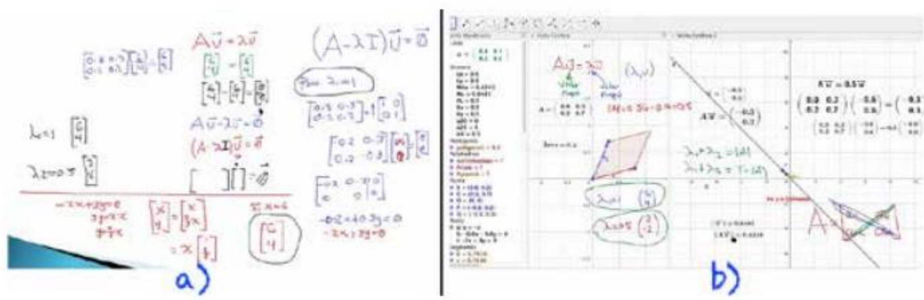

Figure 2a. PowerPoint open and Zoom Annotation tool. / **Figure 2b.** DGE (both as whiteboard).

A didactic setup for this instrumental demonstration-technique orchestration is the sharing of the videoconference screen by the teacher, which allows students to follow the demonstration, similar to a classroom. As a mode of exploitation, the teacher shows the entire class each new command and the functionality of the command on the artifact.

For this experience, the instrumental orchestration demonstration-technique has been modified to what is called Demonstration-Technique-Online, which refers to the demonstration of techniques for the use of an artifact by the teacher, supported by a student-sherpa. The didactic configuration for this orchestration is synchronous videoconference screen sharing by the teacher, and then the intervention of a student-sherpa (under the criterion of working on a computer and having a good internet connection), who reaffirms the teacher's explanation. As a mode of exploitation, the teacher and the student-sherpa show the whole class each new command and its functionality in the artifact. The diagnostic test and the task construction of a quadrilateral are activities to achieve certain instrumentation with the environment. The findings of the different types of instrumental orchestrations observed in the experience, the connection students make with the geometric properties of the vector $\vec{u}$ in $\mathbb{R}^2$ with its linear transformation (matrix) and the mathematical relationship between the vectors $A\vec{u}$ and $\lambda\vec{u}$- eigenvalue and eigenvector are presented.

Relationship of matrix columns to vectors in the DGE in $\mathbb{R}^2$

In this activity the teacher used a demonstration-technical orchestration to indicate to the students that upon opening the EDVI three views were presented: algebraic (red outline), Figure 1 (green outline), and Figure 2 (blue outline) (see Figure 3). They were instructed that this activity would be focused on graphical view 1 (green outline), and were given 20 minutes to interact with the EDVI and, at the same time, respond to the guided exploration sheet. Students worked individually, made the video recording of their activities and added them to the cloud in their respective folders.

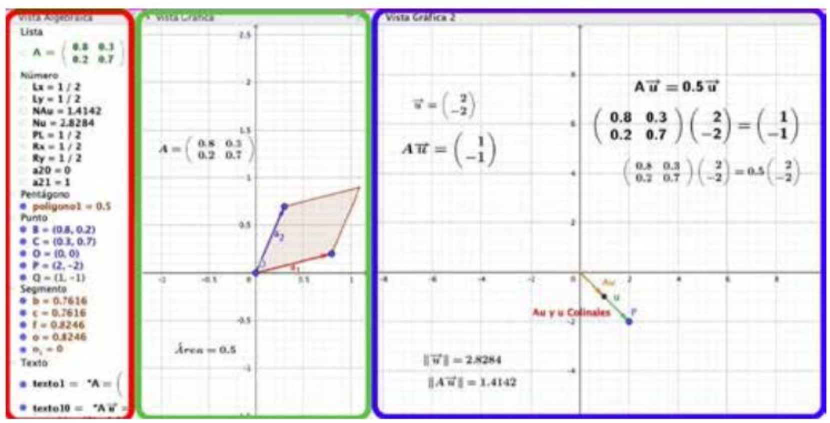

Figure 3. Visualization of the algebraic, Figure 1 and Figure 2 views.

The most used instrumental orchestrations were: support-technical and discuss-the-screen as an orchestration of the whole class. It should be noted that the guided instruction sheet was fundamental to achieve the objective and somehow supplants the orchestration "walk among the computers", when reviewing the work done. Likewise, the didactic performance Nicté-Ha: "I was recording my activity, but it stopped recording by itself, do I start it again or continue recording where I left off?" was presented. As it was intended to observe the students' work, it was indicated that if the recording was interrupted they should try to resume their work at the point where it was interrupted. At the end of the activity, another didactic performance was presented: "Teacher, I have finished, can I upload my work later? It's just that at the moment my internet is slow" (Randy). What is relevant about these didactic performances is that they are only observed in the online modality.

Students recognize that the values of vector $\overrightarrow{\alpha_1}$ and vector $\overrightarrow{\alpha_2}$ correspond to the values of the first and second columns of the matrix (see Figure 3). Then, the teacher continued with the next activity and asked a student:

Teacher: "How are the column vectors of matrix A, are they collinear or not collinear?".

Leonardo: "They are not collinear because in the graphical representation I saw it. Also because there is no 0 or 180 degrees between them".

Teacher: "Are the column vectors of the matrix dependent or independent?".

Arely: "They are dependent" [incorrect answer].

At this point, the teacher changes his orchestration to link-screen-blackboard to connect the geometric interpretation of the determinant of the matrix $A = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$ with the area of the polygon generated by the vectors $\overrightarrow{\alpha_1}$ and $\overrightarrow{\alpha_2}$ and show the students the relationship of the null determinant and the uniqueness of the matrix A. In this activity students first approached the determinant of a 2 x 2 matrix geometrically and recognized that the area generated by *det* (A) defines the area of the parallelogram formed by the vectors $\overrightarrow{\alpha_1}$ and $\overrightarrow{\alpha_2}$.

Recognition of the relationships of the vector $\vec{u}$ and the vector A$\vec{u}$

The objective of this activity was for students to discover the collinearity of the vectors $A\vec{u}$ and $\vec{u}$ and that the proportionality ratio between them is $\lambda = \frac{\|A\vec{u}\|}{\|\vec{u}\|}$, and then to address the equality $A\vec{u} = \lambda\vec{u}$. graphical view 2 (blue outline), in Figure 3, shows two vectors: a vector $\vec{u}$ draggable over that graphical view and its transformed vector $A\vec{u}$. When the vector $\vec{u}$ is dragged, the vector $A\vec{u}$ moves according to the transformation matrix. Graphical view 2 also includes an arithmetic representation $A\vec{u} = \lambda\vec{u}$, the value of the norm of vector $\vec{u}$ and the value of the norm of vector $A\vec{u}$, the measure of the angle between vector $\vec{u}$ and vector $A\vec{u}$ in degrees, as well as a text with the legend "$A\vec{u}$ and $\vec{u}$ are collinear", which is displayed only when these vectors have an error ±0. 01º with the angles of 0º or 180º, to avoid that due to approximation problems the desired solution is never reached. Another arithmetic representation $A\vec{u} = \lambda\vec{u}$ with an approximate λ is also added, which is only seen when the above text is displayed. The student can modify the values of the matrix A.

This activity was performed at home without teacher guidance. Another didactic performance was presented when a student removed the access permission to the Google Drive folder to all users, but the teacher managed to recover the permission for the students; because all users have the edit permission, this kind of problems with the storage of the files can occur. Again, the teacher used instrumental orchestration explain-the-screen through the work of one of the students as a mode of exploitation. The selected student's work was in criteria of the first work that was uploaded to the Google Drive and, after having submitted, the teacher used the discuss-the-screen orchestration.

> Teacher: "Dalia, how did you get that the vector $\vec{u}$ = (4, −4) and $\vec{u}$ = (−4, 4) are collinear to the vector $A\vec{u}$?"
>
> Dalia: "It's just that I kept trying it out".
>
> Teacher: "Did anyone observe that, given a vector $\vec{u}$ collinear to $A\vec{u}$, the vector $-\vec{u}$ was also collinear to $A\vec{u}$?"
>
> Nicole: "I did, teacher."
>
> Teacher: "How did you come to this idea?"
>
> Nicole: "Maybe because they are collinear they must be on the same line. That's why I came to that conclusion in my mind."

Next, the teacher asked the students to drag the vector $\vec{u}$ to the coordinates (-2,2), observe the coordinates of the vector $A\vec{u}$ and calculate $||A\vec{u}||/||\vec{u}||$. Subsequently, let them drag the vector $\vec{u}$ to the coordinates (-1,1), (2,-2) and (4,-4), observe the coordinates of the vector $A\vec{u}$ and calculate $||A\vec{u}||/||\vec{u}||$.

> Teacher: "What could you infer or generalize with these data?".
>
> Arely: "That all those points are collinear. If we were to put them all, they would be a straight line. Therefore, they would be collinear."
>
> Teacher: [plots the straight line x + y = 0] "What could you say?".
>
> Arely: "That, for example, the vectors that you gave, that you underlined, if we were to place them in the Cartesian plane, they would still be on the same line, therefore, those four points (-2,2), (-1,1), (2,-2) and (4,-4) are collinear".
>
> Teacher: "What could we generalize?".
>
> Ulises: "The vector $A\vec{u} = (-1,1)$ if we multiply it by the scalar 2 gives us the vector $\vec{u}$".
>
> Leonardo: "I saw that they were two straight lines".

Up to this point, the students fail to identify what the equations of the lines are, but they already have the idea that all the vectors $\lambda\vec{u}$ that are on them are collinear to the vector $A\vec{u}$. The teacher reaffirmed to them that all the vectors $\vec{u}$ that are on the line were collinear to the vector $A\vec{u}$. In addition, they were asked to calculate $\frac{\|A\vec{u}\|}{\|\vec{u}\|}$ and check that it is always "0.5" for one line and "1" for the other. At this point the teacher commented that they were studying the matrix equation $A\vec{x} = \lambda\vec{x}$, and gives the formal definition of eigenvalue and eigenvector of a matrix . Subsequently, he used the link-screen-whiteboard orchestration to show them again the relationship of the area of the polygon formed by the columns of the matrix and the determinant of the 2 x 2 matrix. He also showed the relationship of the lambdas with the determinant of the matrix A and its trace (see figure 4).

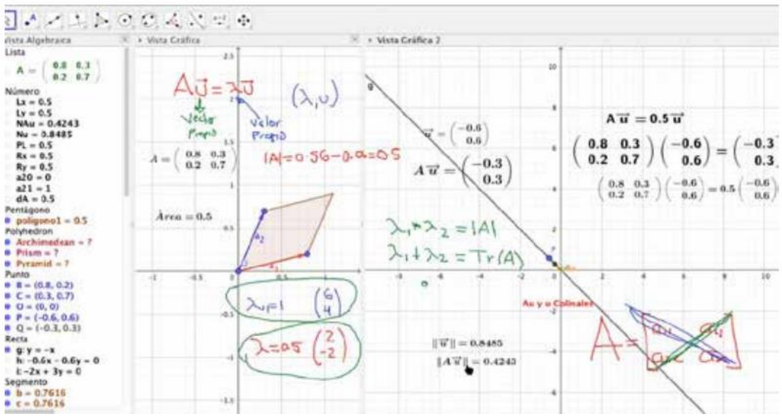

Figure 4. Relationship of the lambdas with the determinant and the trace of the matrix A.

Algebraic calculation of eigenvalues and eigenvectors

Thomas and Stewart (2011) point out that textbooks show $A\vec{x} = \lambda\vec{x}$, then insert a small step to $A\vec{x} = \lambda I\vec{x}$, but do not explain what $\lambda I\vec{x}$ is; i.e., that I is a $\notin \mathbb{R}$.matrix. With Board-instruction-screen orchestration the teacher explains how the equation $A\vec{x} = \lambda\vec{x}$ is transformed into $(A - \lambda I)\vec{x} = \vec{0}$ with student participation (see Figure 5).

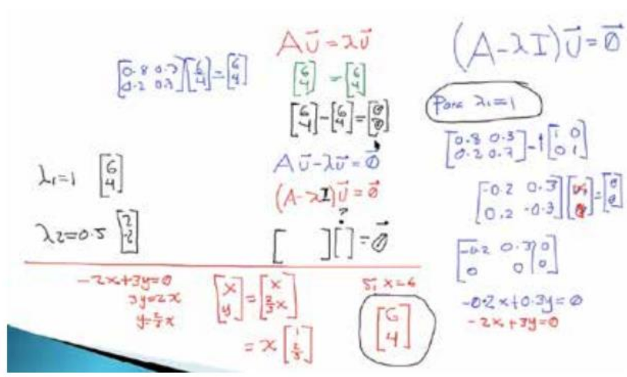

Figure 5. The step from equation $A\vec{x} = \lambda\vec{x}$, to equation $(A - \lambda I)\vec{x} = \vec{0}$.

> Teacher: "Remember that $A - \lambda$ cannot be operated on, what did we do in the topic of operations between matrices so that it could be operated on?".

Leonardo: "It was multiplied by the matrix identity".

The teacher developed the solution of $(A - \lambda I)\vec{x} = \vec{0}$ with $\lambda_1 = 1$ to obtain the eigenspace; after this (-0.2x + 0.3y = 0), he asked a student to multiply the equation of the eigenspace by ten, to note that it was one of the two lines that the student had discussed in the previous class. Subsequently, the teacher asked the students to enter the equation of the line "2x + 3y = 0" in GeoGebra and to place the vector $\vec{u}$ on it, so that they could corroborate that when the vector $\vec{u}$ is on the line, the vector $\vec{u}$ and the vector $A\vec{u}$ are collinear with the same eigenvalue $\lambda_1 = 1$.. Given the eigenspace, the teacher showed the students how to obtain an eigenvector; demonstrated the calculation of eigenvalues and eigenvectors analytically; and corroborated with the students the eigenvalues and eigenvectors they obtained dynamically in the DGE.

Calculation of eigenvalues and eigenvectors of matrices of order 2 and 3.

The teacher instructed them to solve this activity with pencil and paper, without the support of technology, and to videotape their solution. For the first matrix $A = \begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix}$, most students (seven) had no difficulty in correctly calculating the characteristic polynomial, the eigenvalues, and then the eigenvectors; only two had problems with their operational algebra. For the matrix $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$, most students (six) had no difficulty in correctly calculating the characteristic polynomial, the eigenvalues and then the eigenvectors. In this exercise three students presented problems in calculating the null space of the matrix $A - \lambda I$ due to algebraic errors. For the matrix

$$A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$$

only four students correctly performed the process to find the eigenvalues and eigenvectors; however, they had arithmetic errors and only one solved it correctly (see Figure 6). Although matrices of order 3 x 3 were not addressed during the activities, the students who attempted to solve it showed to be able to apply what they learned or performed in $\mathbb{R}^2$ to vectors in $\mathbb{R}^3$.

Figure 6. a) Hand calculation of the first eigenvector, b) calculation of the second eigenvector, and c) calculation of the third eigenvector.

The possibility of dynamically experimenting and geometrically visualizing the effect that a linear transformation or matrix produces on a vector, helped to better understand the relationship $A\vec{x} = \lambda\vec{x}$. The online exposition promoted group work and originated collaborative work by forming small subgroups for discussion and analysis, directed on the platform by the teacher; this allowed the students to solve by themselves a large part of their deficiencies. By having the students' answers to the exercises before class, the teacher worked on redesigning his instruction towards something more adequate.

CONCLUSIONS

Currently, when talking about issues related to the teaching and learning of mathematics, technological tools are unquestionably included. These cannot be thought of as artifacts that are used freely, but their implementation in the classroom has shown that students require a process of appropriation of these tools. This is the great contribution of the instrumental approach, to support mathematics teachers in their efforts to integrate technology into their teaching practice, although it is still a challenge for the mathematics education community.

The research question: what instrumental orchestrations might a university professor choose when using technology in value and eigenvector online teaching framed the present study. A university professor's teaching practice with engineering students during the onset of the pandemic caused by Covid-19 was examined using online teaching through the theoretical framework of instrumental orchestration. In 2004, the educational mathematics community was introduced to the instrumental orchestration metaphor to reflect on the integration of artifacts into their teaching and organization; however, as technology changes, the types of instrumental orchestrations-previously identified-must be reexamined with the possibility of being modified or expanded.

This research proposes to the community the possibility of expanding the previously defined instrumental orchestrations with demonstration-technique-Online, for an online teaching; that is, through the use of a dynamic software, the creation of EDVI and guided exploration sheets, evidences are provided to perform teaching-learning situations of mathematical concepts, such as eigenvalues and eigenvectors, which can be used in the time estimated by the study program and reproducible in any similar situation.

The orchestrations presented in this article originated because the students uploaded video recordings of their work and the teacher had the opportunity to access and make orchestrations from these works. This study contributes to a long way to go. This contribution, up to now, is not conclusive, since more experimentation is still needed; however, it is encouraging to continue with strategies of this type in teaching, which could be covered with new contributions from the research and teaching environment. This proposal does not intend to compete with or replace face-to-face teaching, but to contribute elements to online teaching, which was already in demand before the current pandemic.

Future considerations

The adverse circumstances of a global pandemic, and the economic, social and emotional crisis, affected the student body; during the course sessions some students contracted covid-19 and others suffered the death of family members. In spite of this, the work was carried out successfully, so we consider that our activities in normal situations will have the desired achievement. We believe that our work provides a new form of orchestration and exploitation that our fellow teachers will be able to use, extend or modify and apply to other mathematical concepts.

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